4-2 Videos Guide

4-2a

Exercises:

- Find a power series representation for $f(x) = \frac{1}{1-x}$.
  - $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots$
- Express $\frac{1}{1+x^2}$ as a power series.

4-2b

Exercises:

- Find a power series representation for the function and determine the interval of convergence.
  - $f(x) = \frac{5}{1-4x^2}$
  - $f(x) = \frac{4}{2x+3}$
  - $f(x) = \frac{x}{2x^2+1}$

4-2c

- More power series representations using term-by-term integration
  - $\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$

  - $\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}$

4-2d

Exercises:

- Find a power series representation for the function and determine the interval of convergence.
  - $f(x) = x^2 \tan^{-1}(x^3)$
  - $f(x) = \left(\frac{x}{2-x}\right)^3$

4-2e

Exercise:

- Use a power series to approximate the definite integral to six decimal places.
$$\int_0^{0.2} x \ln(1+x^2) \, dx$$